

Modeling Educational Achievement

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Presentation

- The Achievement Equation
- Example of Use
- Persistence Calculation
- The calculation of Achievement as a double summation.
- Predicted Achievement with changes in 4 Variables
- The timeframe issue
- The pipeline problem
- A mathematical simplification
- Summary

The Equation that Models Achievement

$$\bullet \quad a = p^{(t/us)}$$

- Where
 - **a** = Long term educational goal **A**chievement
 - **p** = Average **P**ersistence (term-to-term) to goal.
 - **t** = **T**otal units needed to achieve goal
 - **u** = **U**nits per term
 - **s** = Passing rate (course **S**uccess)
- t/us is the number of terms students need to achieve their educational goal.

CCSF Students who First Appeared in Credit Academic History 1998 - 2002 (fall and spring starters only) and were tracked through fall 2010

	N	Persistence	Terms	Units Taken	Units per Term	Units Passed	Passing Percent	Units Needed	Terms Needed	Predicted Achievement	Actual Achievement
Total	72,299	80%	3.99	27.45	6.88	19.36	71%	60	12.36	7%	12%

60+ unit Achievement Calculation

- Terms needed = total units / ((units per term)(passing rate))
- = $60 / ((6.88)(.71))$
- = $60 / 4.88$
- = 12.36

- Achievement = $.8^{12.36}$
- = .07

Persistence Calculation

Terms at CCSF	N	Number Remaining	Percent Remaining	Persistence
1	27,450	72,299	100%	100%
2	11,660	44,849	62%	62%
3	6,710	33,189	46%	74%
4	4,801	26,479	37%	80%
5	3,755	21,678	30%	82%
6	3,087	17,923	25%	83%
7	2,708	14,836	21%	83%
8	2,244	12,128	17%	82%
9	1,848	9,884	14%	81%
10	1,581	8,036	11%	81%
11	1,350	6,455	9%	80%
12	1,050	5,105	7%	79%
13	897	4,055	6%	79%
14	672	3,158	4%	78%
15	563	2,486	3%	79%
16	473	1,923	3%	77%
17	346	1,450	2%	75%
18	281	1,104	2%	76%
19	211	823	1%	75%
20	154	612	1%	74%
21	131	458	1%	75%
22	98	327	0%	71%
23	72	229	0%	70%
24	45	157	0%	69%
25	36	112	0%	71%
26	28	76	0%	68%
27	14	48	0%	63%
28	13	34	0%	71%
29	5	21	0%	62%
30	6	16	0%	76%
31	4	10	0%	63%

What's the Average Persistence over 12.36 semesters?

- $P = ((1.00)(.62)(.74)(.8)(.82)(.83)(.83)(.82)(.81)(.81)(.80)(.79))^{(1/12)}$
- $= .8$
- This is the **geometric** Mean.

60 Unit Completion

Round d Units Passed Percent	Interval	N	% of Total	Persist ence	Terms	Units	Units per Term	Units Passed	Passed %	Units Needed	Terms Needed	Predicted Achieve ment	Contrib ution to Total	Actual Achieve ment
0	0-.04	13,791	19%	0%	1.24	5.74	4.62	0.01	0%	60	8639.94	0%	0%	0%
0.1	.05-.14	1,149	2%	0%	3.27	22.91	7	2.39	10%	60	82.13	0%	0%	0%
0.2	.15-.24	1,507	2%	0%	3.93	28.1	7.15	5.6	20%	60	42.13	0%	0%	0%
0.3	.25-.34	2,944	4%	0%	3.86	26.43	6.85	7.93	30%	60	29.19	0%	0%	0%
0.4	.35-.44	2,678	4%	81%	5.23	38.06	7.27	15.39	40%	60	20.4	1%	0%	3%
0.5	.45-.54	4,824	7%	82%	4.66	32.78	7.04	16.45	50%	60	16.98	4%	0%	7%
0.6	.55-.64	3,947	5%	86%	6.78	50.83	7.5	30.6	60%	60	13.29	14%	1%	18%
0.7	.65-.74	4,850	7%	87%	6.89	51.31	7.44	35.88	70%	60	11.53	21%	1%	25%
0.8	.75-.84	5,742	8%	88%	7.04	51.74	7.35	41.32	80%	60	10.22	26%	2%	30%
0.9	.85-.94	5,284	7%	91%	8.09	62.39	7.72	56.21	90%	60	8.63	46%	3%	44%
1	.95-1.0	25,583	35%	73%	2.77	16.38	5.9	16.22	99%	60	10.26	4%	1%	9%
Grand Total		72,299	100 %	80%	3.99	27.45	6.88	19.36	71%	60	12.36	7%	9%	12%

For Completely Successful Students by Rounded Units Taken Per Term

Units Per Term	Interval	N	% of Total	Persistence	Terms	Units	Units per Term	Units Passed	Passed %	Units Needed	Terms Needed	Predicted Achievement	Contribution to Total	Actual Achievement
0	0-2.5	3,685	14%	0%	1.83	2.27	1.24	2.27	100%	60	48.4	0%	0%	0%
4	2.6-6.5	14,254	56%	75%	2.41	8.49	3.52	8.46	100%	60	17.09	1%	0%	1%
8	6.6-10.5	4,196	16%	81%	3.99	31.3	7.85	30.93	99%	60	7.74	20%	3%	20%
12	10.6-14.5	2,785	11%	85%	4.14	47.5	11.48	46.94	99%	60	5.29	42%	5%	40%
16	14.6 -18.5	651	3%	67%	2.14	32.8	15.29	32.51	99%	60	3.95	21%	1%	22%
Grand Total		25,583	100%	73%	2.77	16.4	5.9	16.22	99%	60	10.26	4%	9%	9%

And the Equation for 60 unit Completion?

- 1 20
- $A = \sum_{i=0} \sum_{j=0} (p_{ij}^{(t/u, s_{ij})}) (n_{ij}/n)$
- $i=0 \quad j=0$

- Where i refers to success and j refers to units per term

English Transfer Course Completion by Starting Level (2003-2006 Starters tracked through fall 2010)

English Students Starting 2003-2006	Levels Below Transfer	N	Persistence	Terms	Units	Units per Term	Units Passed	Passed %	Units Needed	Terms Needed	Predicted Achievement	Actual Achievement
English 90	4	1,889	66%	3.23	9.79	3.03	5.62	57%	15	8.62	3%	19%
English 92	3	1,141	71%	3.25	9.76	3	5.97	61%	12	6.54	11%	30%
English 93	2	2,830	66%	2.47	7.41	3	5.1	69%	9	4.36	16%	38%
English 96	1	2,210	60%	2.02	6.06	3	3.87	64%	6	3.13	20%	52%

Observed Versus Predicted Achievement in the English Sequence 2003-2006 starters followed through fall 2010

	Levels Below Transfer	Transfer Course completion	
Level		Actual	Predicted
90	4	19%	19%
92	3	30%	30%
93	2	38%	38%
96	1	52%	53%

Mathematics Starting Fall 2004 – Spring 2006 Followed to Spring 2011

Math Course	N	Terms	Units	Units Per Term	Units Passed	Passing Rate	Persistence	Units Needed	Terms Needed	Predicted Achievement	Actual Achievement	log a/ log p
E3- Arithmetic	1021	2.68	8.40	3.13	4.10	49%	65%	13	8.51	3%	6%	1.54
840 - Elementary Algebra	1481	2.51	8.50	3.38	4.72	56%	67%	10	5.33	12%	17%	1.74
860 - Intermediate Algebra	1035	2.58	9.92	3.84	5.53	56%	73%	7	3.27	35%	40%	2.17

Observed Versus Predicted Achievement in the Math Sequence

Levels Below Transfer	Math Course	Transfer Course completion	
		Actual	Predicted
3	E3	6%	7%
2	840	17%	19%
1	860	40%	43%

Completion of 60 Units

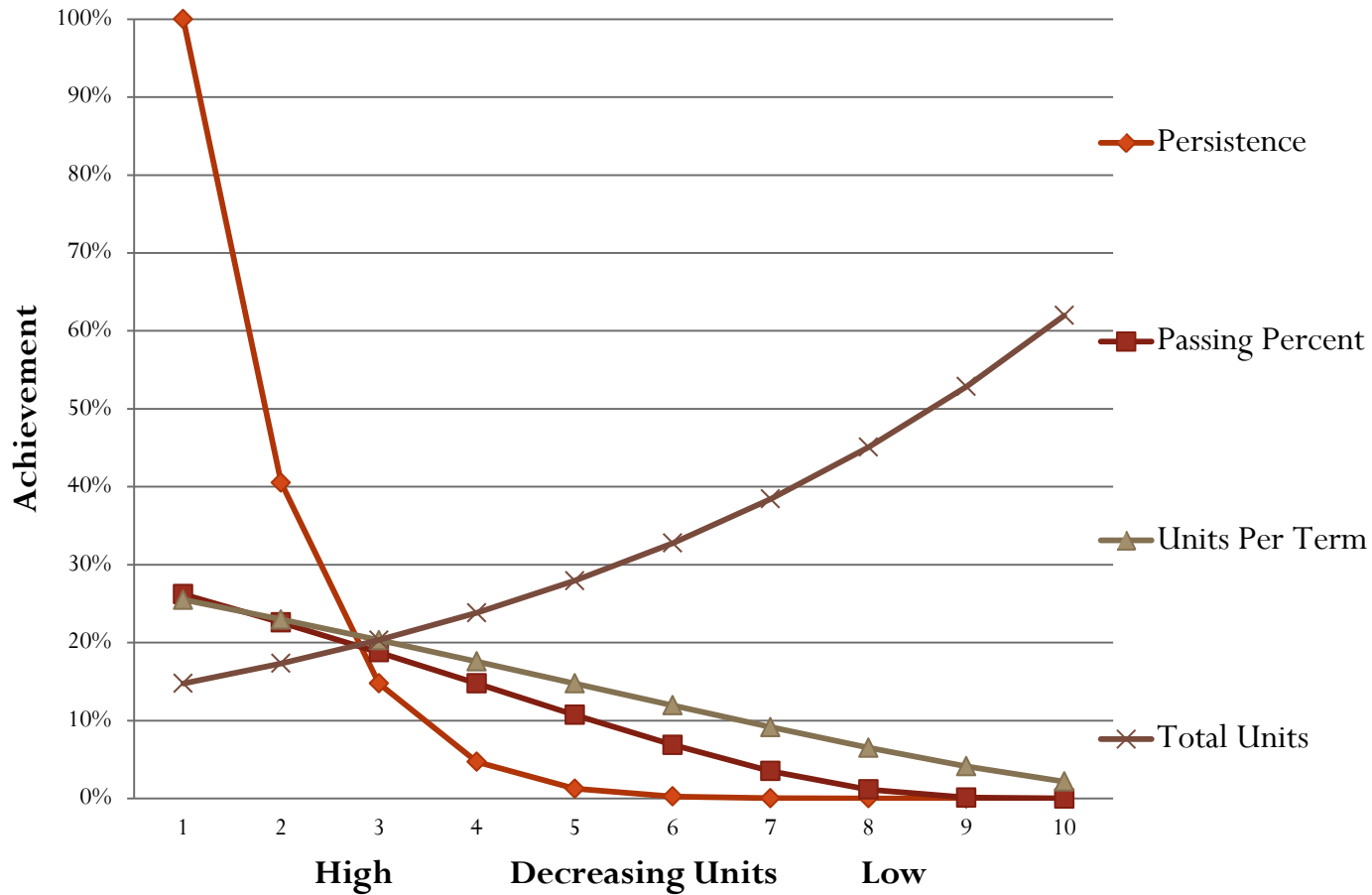
New Students at CCSF By Academic Year

	Starting Year of Credit CCSF Students					
Data	1987	1990	1993	1997	2000	2003
N	16,901	16,423	12,292	15,922	17,179	15337
Terms	4.27	4.36	4.64	4.20	4.00	3.89
Units	27.44	29.29	33.14	28.35	26.89	28.20
Units Passed	18.29	19.80	23.12	19.68	18.97	19.78
UnitsPerTerm	6.43	6.72	7.14	6.75	6.73	7.24
Passing %	67%	68%	70%	69%	71%	70%
Terms to 60	13.99	13.20	12.04	12.80	12.64	11.81
Persistence	81%	81%	82%	81%	80%	80%
Predicted to60	5%	7%	9%	6%	6%	7%
Actual to 60	11%	13%	15%	12%	12%	12%

Predicted Achievement with changes in Persistence - $a = p^{(t/us)}$

Total Units	Persistence	Units per Term	Units Passed Percent	Terms Needed	Predicted Achievement
60	100%	10	70%	8.57	100%
60	90%	10	70%	8.57	41%
60	80%	10	70%	8.57	15%
60	70%	10	70%	8.57	5%
60	60%	10	70%	8.57	1%
60	50%	10	70%	8.57	0%
60	40%	10	70%	8.57	0%
60	30%	10	70%	8.57	0%
60	20%	10	70%	8.57	0%
60	10%	10	70%	8.57	0%

Predicted Achievement for All Factors Together



The Timeframe Issue

- $a = p^{t/us}$
- $\text{Log } a = \log p^{t/us}$
- $\text{Log } a = (t/us) \log p$
- $\text{Log } a / \log p = t/us$

- What's $\log a / \log p$?

Log a / Log p

- Terms Needed = $\text{Log } a / \text{Log } p$
- **IMPLICATION:** You can figure out the terms you have available in order to obtain a desired level of achievement by **ONLY KNOWING** the Persistence! The educational goal (Phd, 6 unit certificate, remedial sequence completion), units per term, and passing rate are not factors.

Log a ÷ Log p

		Persistence										
			1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
		Log	0	-0.05	-0.10	-0.15	-0.22	-0.30	-0.40	-0.52	-0.70	-1.00
Achievement	1	0.00										
	0.9	-0.05		1.00								
	0.8	-0.10		2.12	1.00							
	0.7	-0.15		3.39	1.60	1.00						
	0.6	-0.22		4.85	2.29	1.43	1.00					
	0.5	-0.30		6.58	3.11	1.94	1.36	1.00				
	0.4	-0.40		8.70	4.11	2.57	1.79	1.32	1.00			
	0.3	-0.52		11.43	5.40	3.38	2.36	1.74	1.31	1.00		
	0.2	-0.70		15.28	7.21	4.51	3.15	2.32	1.76	1.34	1.00	
0.1	-1.00		21.85	10.32	6.46	4.51	3.32	2.51	1.91	1.43	1.00	

$$a = p^{(t/us)}$$

The Pipeline Problem –

Total Population as a Multiple of the New Student Population

Term	New	Total	Multiple
Fall 2008	10,333	36,360	3.52
Spring 2009	7,677	37,813	4.93
Fall 2009	9,824	36,756	3.74
Spring 2010	6,290	36,974	5.88
Fall 2010	8,005	34,855	4.35
Spring 2011	6,931	37,568	5.42
Total	49,060	220,326	4.49

Enrollment Metric

Total Enrollment = new students +

Remaining students who started prior term +

Remaining students who started two terms ago +

Remaining students who started three terms ago +

....

$$= a_0p_0^0 + a_1p_1^1 + a_2p_2^2 + \dots + a_kp_k^k$$

where

a = new student enrollment

p = persistence rate.

Simplifying Assumptions

- Assume:
- $a_0 = a_1 = a_2 = \dots = a$
- $p_0 = p_1 = p_2 = \dots = p$
- The new student enrollment number and the persistence rate are the same across groups.
- Total enrollment = $a \sum_{i=0}^k p^i$

The relationship between Persistence and Total Enrollment

Persistence Rates

Terms before	60%	70%	80%	90%
0	1.00	1.00	1.00	1.00
1	0.60	0.70	0.80	0.90
2	0.36	0.49	0.64	0.81
3	0.22	0.34	0.51	0.73
4	0.13	0.24	0.41	0.66
5	0.08	0.17	0.33	0.59
6	0.05	0.12	0.26	0.53
7	0.03	0.08	0.21	0.48
8	0.02	0.06	0.17	0.43
9	0.01	0.04	0.13	0.39
10	0.01	0.03	0.11	0.35
11	0.00	0.02	0.09	0.31
12	0.00	0.01	0.07	0.28
Sum	2.50	3.30	4.73	7.46

Enrollment and Persistence

- Total enrollment = $a \sum_{i=0} p^i$
- $36,000 = (8,000)(4.50)$
- If the persistence rate were 7.50 then
- $60,000 = (8,000)(7.50)$
- And
- $60,000 / 36,000 = 1.67$
- **AN ENROLLMENT INCREASE OF 67%! From a persistence increase of 10%!**

And if Enrollment cannot be increased?

- $36,000 = ???(8,000)*(4.50)(7.50/4.50)$
- $= 4.50/7.50(8,000)(4.50)(7.50/4.50)$
- $36,000 = (4,800)(7.50)$
- A reduction in access of $1 - 4,800/8,000 = (40\%)$
- And $a =$ access through new students served
- $\sum p^i =$ achievement (through persistence)
- **INCREASED ACCESS AND ACHIEVEMENT ARE INCOMPATIBLE GOALS IN A FIXED ENROLLMENT ENVIRONMENT!**

A mathematical Simplification

- $P = ((n_1/n_1)(n_2/n_1)(n_3/n_2)(n_4/n_3)\dots(n_k/n_{k-1}))^{(1/k)}$
- $P = (n_k/n_1)^{(1/k)}$
- $a = p^{t/us}$
- $t/us = k$
- $a = ((n_k/n_1)^{(1/k)})^{(k)}$
- $a = n_k/n_1$
- **ACHIEVEMENT IS THE NUMBER OF STUDENTS REMAINING AT THE COMPLETION TERM AS A FRACTION OF THOSE WHO STARTED!**

$$a = \sum_{i=0}^1 \sum_{j=1}^{18} (p_{ij}^{(t/u s)}) (n_{ij}/n_1)$$

$$a = \sum_{i=0}^1 \sum_{j=1}^{18} (n_{kij} / n_{1ij}) (n_{1ij} / n_1)$$

$$a = \sum_{i=0}^1 \sum_{j=1}^{18} (n_{kij} / n_1)$$

$$a = (n_{k11+} + n_{k12+...+} + n_{k21+} + n_{k22+...+} + n_{kij}) / n_1$$

In Sum

- Achievement can be modeled with an exponential equation.
- Persistence is the most important variable because achievement changes most rapidly with changes to it.
- Without increases to persistence the timeframe for achievement is so narrow that no meaningful improvement can be made.
- Increases to persistence either overwhelm scarce resources, or squeeze out the access of new students.

Elementary Algebra 2004-2006 Starters followed through Fall 2011

Rounded Passing Rate	N	% of Total	Terms	Units	Units Per Term	Units Passed	Passed%	Persistence	Units Needed	Terms Needed	Perdicted Achievement	Contribution	Actual Achievement
0	423	29%	1.42	4.33	3.05	-	-	0	10	∞	0%	0%	0%
0.1	7	0%	6.29	23.42	3.72	3.00	0.13	0	10	20.97	0%	0%	0%
0.2	46	3%	5.40	19.30	3.57	3.72	0.19	0	10	14.50	0%	0%	0%
0.3	63	4%	4.18	14.33	3.43	4.28	0.30	0	10	9.75	0%	0%	2%
0.4	96	6%	3.74	12.97	3.47	5.19	0.40	77%	10	7.20	15%	1%	13%
0.5	102	7%	3.65	12.43	3.41	6.19	0.50	77%	10	5.90	21%	1%	13%
0.6	85	6%	4.11	14.34	3.49	8.66	0.60	84%	10	4.75	43%	2%	46%
0.7	60	4%	4.29	15.23	3.55	10.81	0.71	94%	10	3.97	77%	3%	60%
0.8	30	2%	5.36	19.48	3.63	15.58	0.80	100%	10	3.44	100%	2%	97%
0.9	9	1%	6.63	24.44	3.69	21.25	0.87	94%	10	3.12	83%	1%	100%
1	560	38%	1.80	5.98	3.32	5.98	1.00	61%	10	3.01	22%	8%	20%
Grand Total	1481	100%	4.26	15.11	3.55	7.70	0.51	65%	10	5.54	10%	19%	17%

English 90 (4 levels below transfer) Achievement – Variables Summed Over Passing Rates

Rounded Units Passed Percent	N	% of Total	Persistence	Terms	Units	Units per Term	Units Passed	Passed%	Units Needed	Terms Needed	Predicted Achievement	Contribution to Total	Actual Achievement
0	629	33%	0%	1.33	4.14	3.10	0.00	0%	15.00	99999.00	0%	0%	0%
0.1	6	0%	0%	7.44	22.33	3.00	3.00	13%	15.00	37.22	0%	0%	0%
0.2	15	1%	0%	5.80	17.70	3.05	3.50	20%	15.00	24.86	0%	0%	0%
0.3	124	7%	0%	4.53	13.75	3.04	4.24	31%	15.00	16.03	0%	0%	0%
0.4	63	3%	71%	6.77	20.53	3.03	8.41	41%	15.00	12.07	2%	0%	3%
0.5	183	10%	73%	3.58	10.81	3.02	5.40	50%	15.00	9.94	4%	0%	4%
0.6	90	5%	89%	6.70	20.12	3.00	11.98	60%	15.00	8.38	39%	2%	41%
0.7	141	7%	86%	4.94	14.92	3.02	10.28	69%	15.00	7.22	35%	3%	36%
0.8	177	9%	91%	5.30	15.97	3.01	12.95	81%	15.00	6.14	56%	5%	58%
1	461	24%	81%	2.75	8.24	3.00	8.24	100%	15.00	5.00	34%	8%	34%
Grand Total	1889	100%	66%	3.23	9.79	3.03	5.62	57%	15.00	8.62	3%	19%	19%

Equation for English with Fixed Units per Term

- 1
- $a = \sum_{i=0} (p_i^{(t/us_i)})(n_i/n)$
- $i=0$
- Where n is the number of students total
- And n_i is the number of students at the particular success level.